

# Fluctuation-induced magnetization dynamics and criticality at the interface of a topological insulator with a magnetically ordered layer

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We consider a theory for a two-dimensional interacting conduction electron system with strong spin-orbit coupling on the interface between a topological insulator and the magnetic (ferromagnetic or antiferromagnetic) layer. For the ferromagnetic case we derive the Landau-Lifshitz equation, which features a contribution proportional to a fluctuation-induced electric field obtained by computing the topological (Chern-Simons) contribution from the vacuum polarization. We also show that fermionic quantum fluctuations reduce the critical temperature  $\tilde{T}_c$  at the interface relative to the critical temperature  $T_c$  of the bulk, so that in the interval  $\tilde{T}_c \leq T < T_c$  is possible to have coexistence of gapless Dirac fermions at the interface with a ferromagnetically ordered layer. For the case of an antiferromagnetic layer on a topological insulator substrate, we show that a second-order quantum phase transition occurs at the interface, and compute the corresponding critical exponents. In particular, we show that the electrons at the interface acquire an anomalous dimension at criticality. The critical behavior of the Néel order parameter is anisotropic and features large anomalous dimensions for both the longitudinal and transversal fluctuations.

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A spin current may exhibit interesting topological properties in systems where a Berry curvature in Bloch momentum space is induced by the underlying band structure [1, 2], like for example in the case of some hole-doped semi-conductors described by a Luttinger Hamiltonian [3] or systems featuring a Rashba spin-orbit coupling [4]. More recent prominent examples involve the quantum spin Hall insulators or topological insulators (TI) [5, 6], where a Berry curvature in momentum space also arises. Depending on the physical situation the Berry curvature may be Abelian or non-Abelian, and determines a magnetic monopole in momentum space.

The surface of a TI, when in contact with a material exhibiting magnetic order, offers a framework for many topological effects. For instance, the two-dimensional system represented by the surface of a topological insulator can be used as substrate for a magnetic layer, which can be either ferromagnetic (FM) or antiferromagnetic (AF). For a FM layer having a TI as substrate, a theoretical study of the magnetization dynamics was carried out recently [7]. In a similar context, the electric charging of magnetic textures has also been discussed [8]. Other interesting electromagnetic topological effects with a similar setup were studied [9–12] and have shown to exhibit properties similar to those of axion electrodynamics [13]. In the axion electrodynamics a topological term of the form  $(8\pi^2)^{-1}\alpha\theta\mathbf{E} \cdot \mathbf{B}$  is present [10, 13] in the action, where  $\alpha$  is the fine structure constant and  $\theta$  is the so called axion field. For the case where  $\theta$  is uniform, time-reversal invariant TIs require  $\theta = \pi$  [10]. Such a term should play a very important role at interfaces of TIs with other insulators. For a magnetic insulating layer on the surface of a TI, a modification of the magnetization dynamics occurs, due to a direct coupling of the magnetization to the electric field. Indeed, we have  $\mathbf{E} \cdot \mathbf{B} = \mathbf{E} \cdot (\mathbf{H} + 4\pi\mathbf{M})$ , giving rise to a magnetoelectric effect, which influences the precession of the magnetization [9].

At the same time, the experimental situation is far from be-

ing clear, as many theoretical predictions have not been confirmed yet. Moreover, in some cases, it seems that the theoretical expectations are not fulfilled. For instance, from a theory perspective one would expect that the coupling of a TI to a FM layer makes the surface states gapped. However, in a very recent experiment [14] where Fe impurities were deposited on  $\text{Bi}_2\text{Se}_3$ , no sign for a gap has been found, in apparent conflict with theoretical expectations. Therefore, further theoretical studies on the coupling of a TI substrate to magnetic system are necessary.

In this paper we consider quantum fluctuations effects stemming from the proximity-induced magnetism on the surface of a TI. We assume that the electrons on the surface of the TI interact via a long-range Coulomb interaction. For the case of a FM layer in contact with the TI, we will derive a Landau-Lifshitz (LL) equation which accounts for these interaction effects. In our calculation an axion-like term emerges due to quantum fluctuations. At the interface, it manifests itself as a Chern-Simons (CS) term [15], which breaks time-reversal symmetry, as a consequence of the coupling of the surface of the TI to the magnetic layer. Furthermore, the electronic quantum fluctuations make the stiffness anisotropic, even if the bulk of the FM layer features an isotropic stiffness. We also show that due to the quantum fluctuations of the electrons, the critical temperature  $\tilde{T}_c$  at the interface is reduced relative to the critical temperature  $T_c$  of the FM layer. This allows the existence of gapless fermions at the interface at the temperature range  $\tilde{T}_c \leq T < T_c$  where the bulk magnetic layer is still magnetically ordered.

It has been shown in a recent study [16] that the best candidate material to gap the topological surface states of a TI is MnSe, which is an AF insulator. Therefore, the study of proximity-induced AF order on a TI is also of great relevance. For such a case we will show that a second-order quantum phase transition occurs at the interface, and that it defines a new universality class. One consequence of this interface

quantum criticality is that the surface electrons become gapless at the quantum critical point (QCP). This does not happen in the FM case we study, where the electrons are always gapped at zero temperature. Hence, the topologically protected gapless modes can be restored at zero temperature by disordering the AF long-range order at the interface. A further important feature of this interface quantum criticality is the emergence of a large anomalous dimension for the Néel order parameter. Interestingly, at the QCP the fermions will also be shown an anomalous dimension. Large anomalous dimensions are known to occur in systems exhibiting phase transitions that do not fit the Landau-Ginzburg-Wilson paradigm, like in the case of the so called deconfined quantum criticality scenario in some Mott insulators [17]. In this context, large anomalous dimensions in theories involving Dirac fermions have also been discussed [18, 19].

Our starting point is the Lagrangian for conduction electrons interacting via a Coulomb interaction on the surface of an insulator either in contact with a bulk FM composed of several layers, similarly to Ref. [7], or with an AF bulk system. Thus, if  $\mathbf{n}$  is the induced magnetization at the interface and  $\mathbf{L}$  the angular momentum, the spin of the conduction electrons,  $\mathbf{S} = (1/2)c^\dagger \vec{\sigma} c$ , is coupled to the total magnetization  $(\mu_B/2)\mathbf{L} + \mathbf{n}$  via an exchange term  $-2J_0\mathbf{S} \cdot [(\mu_B/2)\mathbf{L} + \mathbf{n}/N_L]$ , where  $c^\dagger = [c_\uparrow^\dagger \ c_\downarrow^\dagger]$ , with  $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  being the Pauli matrices, and  $N_L$  is the number of magnetic insulating layers. The lack of inversion symmetry in the direction perpendicular to the interface leads to a spin-orbit coupling of the Rashba type. Thus, the Lagrangian for the conduction electrons at the interface reads (we are assuming units where  $\hbar = c = 1$ ),

$$\mathcal{L}_c = c^\dagger [i\partial_t + \mu + e\varphi - iv(\sigma_y\partial_x - \sigma_x\partial_y) + J\vec{\sigma} \cdot \mathbf{n}]c - \frac{\epsilon}{4\pi}\varphi|\nabla|\varphi, \quad (1)$$

where  $v \propto J_0$  and  $J = J_0/N_L$ . In writing the above Lagrangian we have assumed that the spin-orbit coupling is much stronger than the usual kinetic term of the conduction electrons, which has been neglected. The auxiliary (Hubbard-Stratonovich) field  $\varphi$  accounts for the Coulomb interaction. Upon integrating out  $\varphi$  the usual Coulomb interaction between the electrons is obtained. The non-local Gaussian term for  $\varphi$  reflects the three-dimensional character of the Coulomb interaction in a two-dimensional problem, similarly to graphene [20]. In this term  $\epsilon$  represents the dielectric constant.

The full Lagrangian of the systems includes the Lagrangian describing the magnetization dynamics, which for a FM system includes a Landau-Ginzburg (LG) type functional and is given by

$$\mathcal{L}_{\text{FM}} = \mathbf{b} \cdot \partial_t \mathbf{n} - \frac{\kappa}{2}(\nabla \mathbf{n})^2 - \frac{m^2}{2}\mathbf{n}^2 - \frac{u}{4!}(\mathbf{n}^2)^2, \quad (2)$$

where  $\kappa, u > 0$  and  $m^2 = a_0(T - T_0)$ , with  $T_0$  being the (mean-field) critical temperature to disorder the FM.  $\mathbf{b}$  is the Berry connection, which fulfills the usual monopole condition,  $\partial b_i/\partial n_j - \partial b_j/\partial n_i = \epsilon_{ijk}n_k/\mathbf{n}^2$ . For  $m^2 < 0$  (or  $T < T_0$ ) the bulk FM is in a ferromagnetically ordered state.

Before considering the magnetization dynamics, let us first consider a fluctuation-corrected mean-field theory where  $\sigma$  is assumed to be uniform, and that the transverse fluctuations vanish. We also neglect the fluctuation effects of the Coulomb interaction. The calculations are done in imaginary time and finite temperature.

In this case, after integrating out the fermions, we obtain the free energy density,

$$\mathcal{F} = -T \sum_{n=-\infty}^{\infty} \int \frac{d^2 p}{(2\pi)^2} \ln(\omega_n^2 + v^2 \mathbf{p}^2 + J^2 \sigma^2) + \frac{m^2}{2} \sigma^2 + \frac{u}{4!} \sigma^4, \quad (3)$$

where  $\omega_n = (2n + 1)\pi T$  is the fermionic Matsubara frequency. After performing the Matsubara sum, the remaining integral over momenta contains a zero temperature contribution which is divergent, requiring regularization and renormalization. Using an ultraviolet cutoff  $\Lambda \sim a^{-1}$ , where  $a$  is the lattice constant, we can cancel the dependence on the cutoff by minimally absorbing it in a redefinition of the Curie temperature of the bulk FM precisely at the interface. The physical requirement (or renormalization condition) is that the zero temperature fermionic gap,  $m_\psi \equiv J\sigma_0$ , is finite in the long-wavelength limit.

The saddle-point approximation yields,

$$a_0(T_c - T) = \frac{u}{6} \sigma^2 + \frac{J^2 T}{\pi v^2} \ln \left[ 2 \cosh \left( \frac{J\sigma}{2T} \right) \right], \quad (4)$$

where  $T_c$  is the renormalized Curie temperature of the bulk FM at the interface. The critical temperature,  $\tilde{T}_c$ , at the interface is obtained by demanding that  $\sigma$  vanishes at  $T = \tilde{T}_c$ . This yields  $\tilde{T}_c = T_c[1 + J^2 \ln 2/(\pi a_0 v^2)]^{-1}$ . On the other hand, by setting  $T = 0$  in Eq. (4), we obtain that *at the interface*  $T_c = um_\psi^2/(6J^2) + J^2 m_\psi/(2\pi a_0 v^2)$ . Note that this expression makes only sense at the interface and does not correspond to the physical critical temperature there, which is actually given by  $\tilde{T}_c$ . Furthermore, since  $v \propto J$  and at leading order  $\sigma_0^2 \approx 6a_0 T_0/u$ , we obtain that  $T_c \rightarrow T_0$  as  $v \rightarrow 0$ . In order to estimate  $\tilde{T}_c$ , we assume that  $T_c \gg um_\psi^2/(6J^2 a_0)$ , such that we have approximately,  $\tilde{T}_c \approx m_\psi T_c [m_\psi + (2 \ln 2)T_c]^{-1}$ . If we use the estimates  $m_\psi \approx 28.2$  meV and  $T_c \approx 70$  K [16, 21], we obtain  $\tilde{T}_c \approx 54$  K. Thus, our fluctuation-corrected mean-field theory implies that the critical temperature at the interface is smaller than the Curie temperature of the bulk FM. Therefore, it is possible to destroy the proximity-induced magnetization at the interface while the bulk FM is still ordered. This occurs typically in a temperature window  $\tilde{T}_c \leq T < T_c$ , where we are assuming that  $T_c$  does not differ appreciably from  $T_0$ . The reduction of the critical temperature at the interface with respect to the bulk one is an important consequence of the interplay between ferromagnetic proximity effect and the spin-orbit coupling.

The next step is to compute the fluctuations of the order parameter around the mean-field theory. Since we are interested in deriving a differential equation for the magnetization dynamics, we will return to real time in the following

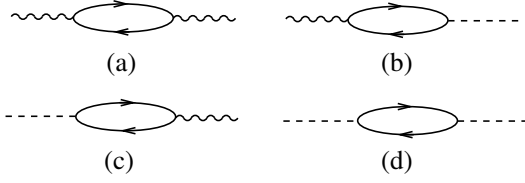


FIG. 1. Diagrams contributing to  $\delta S_{\text{gauge}}$ , Eq. (7). The wiggled line represents the vector field  $a_\mu$ , the solid line is a Dirac fermion, and the dashed line represents the fluctuating part of the  $\sigma$  field. The diagrams (b) and (c) cancel out.

and essentially consider a zero temperature calculation. In order to facilitate our analysis of the problem, it is convenient to rewrite the Lagrangian for the conduction electrons in a QED-like form, which is achieved by the rescalings  $\varphi \rightarrow (J/e)\varphi$ ,  $x_i \rightarrow vx_i$  ( $i = 1, 2$ ), in the action, to obtain

$$\mathcal{L}_c = \bar{\psi}(i\partial - J\phi)\psi + J\sigma\bar{\psi}\psi - \frac{\zeta}{2}a_0|\nabla|a_0, \quad (5)$$

where the Dirac matrices are defined by  $\gamma^0 = \sigma_z$ ,  $\gamma^1 = -i\sigma_x$ , and  $\gamma^2 = i\sigma_y$ ,  $\psi = vc$ , such that the usual relativistic notations for spinors hold with  $\bar{\psi} = \psi^\dagger\gamma^0$ , and also the usual Dirac slash notation,  $\not{Q} = \gamma^\mu Q_\mu$ , is being used. The gauge field is given by  $a^\mu = (\varphi, n_y, -n_x)$  and  $\sigma = n_z$ , and the dielectric constant,  $\zeta \equiv \epsilon v^2 J^2 / (2\pi e^2)$ . The chemical potential was absorbed in  $a^0$ . Note the different physical nature of the components of  $a^\mu$ , with the zeroth component corresponding to the scalar potential, and the spatial components being related to the transverse components of the magnetization relative to the  $z$ -direction.

The main advantage of the QED-like representation of the Lagrangian for the conduction electrons is that it allows us to integrate out the fermions using standard field theoretic techniques. In order to better explore different approximation schemes, we will generalize our original Lagrangian to the case where there are  $N$  fermionic orbital degrees of freedom. Thus, integrating out the fermions yields the gauge-invariant contribution to the effective action,

$$S_{\text{gauge}} = iN\text{Tr} \ln(i\partial - J\phi + J\sigma). \quad (6)$$

The lowest order diagrams associated to the fluctuating fields from Eq. (6) are shown in Fig. 1. Approximate evaluation of  $S_{\text{gauge}}$  at long wavelengths yields the leading fluctuation contribution,  $S_{\text{gauge}} \approx S_{\text{eff}}^{\text{MF}} + \delta S_{\text{gauge}}$ , where

$$\delta S_{\text{gauge}} = \frac{NJ^2}{8\pi} \int d^3x \left\{ -\frac{1}{3m_\psi} (\epsilon_{\mu\nu\lambda} \partial^\nu a^\lambda)^2 + \epsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda + \frac{1}{m_\psi} [(\partial_t \tilde{\sigma})^2 - (\nabla \tilde{\sigma})^2] \right\}, \quad (7)$$

with the fluctuation  $\tilde{\sigma}$  arising from the decomposition  $\sigma = \langle \sigma \rangle + \tilde{\sigma}$ . The quadratic fluctuation term in  $\tilde{\sigma}$  will generate an anisotropy in the magnetic system, which is isotropic in the bulk. The first term in Eq. (7) corresponds to a Maxwell term in  $(2+1)$ -dimensional electrodynamics. The second term is a CS term [15] generated by the quantum fluctuations. This

CS term reflects the breaking of time-reversal symmetry due to the coupling to a magnetic layer. In order to better appreciate the effect of the CS term, it is useful to rewrite the CS contribution to  $S_{\text{gauge}}$  in the form,

$$S_{\text{CS}} = \frac{\sigma_{xy}}{4\pi} \int d^3x (n_y \partial_t n_x - n_x \partial_t n_y + 2\mathbf{n} \cdot \nabla \varphi), \quad (8)$$

where  $\sigma_{xy} = \sigma_{xy}^0 NJ^2 / e^2$ , with  $\sigma_{xy}^0 = e^2 / 2$  (in units where  $\hbar = 1$ ), is the induced Hall conductivity. It is readily seen that the contribution proportional to  $n_x \partial_t n_y - n_y \partial_t n_x$  yields an additional Berry phase, as discussed previously in Ref. [7]. The term proportional to  $\mathbf{n} \cdot \nabla \varphi$  is a crucial contribution stemming from the Coulomb interaction between the fermions at the interface. Indeed, since  $\varphi$  is a fluctuating scalar potential associated to the Coulomb interaction, this term yields a contribution proportional to  $\mathbf{M} \cdot \mathbf{E}$ , where  $\mathbf{E} = -\nabla \varphi$  is a fluctuation-induced electric field, and  $\mathbf{M} \sim \mathbf{n}$ . Thus, this term corresponds to an emergent axion-like term.

From the CS term we can also determine the topological current,  $K^\mu = (4\pi)^{-1} \sigma_{xy} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$ , which up to the prefactor  $\sigma_{xy} / (4\pi)$  has components  $(\nabla \cdot \mathbf{n}, \partial_t n_x - \partial_y \varphi, \partial_t n_y + \partial_x \varphi)$ . The topological current is identically conserved, i.e.,  $\partial_\mu K^\mu = 0$ .

The Landau-Lifshitz equation for the magnetization dynamics at the interface can now be obtained by extremizing the effective action (note that here  $\nabla \cdot \mathbf{n} = \partial_x n_x + \partial_y n_y$ , since there is no  $z$ -component for the  $\nabla$  operator in the present two-dimensional problem),

$$\partial_t \mathbf{n} = \mathbf{n} \times \left\{ \rho_s^{ij} (\nabla^2 \mathbf{n})_j \mathbf{e}_i + \frac{Z\sigma_{xy}}{2\pi v^2 m_\psi} [(\partial_t^2 \mathbf{n})_z \mathbf{e}_z - \nabla(\nabla \cdot \mathbf{n})] \right\} + \frac{Z\sigma_{xy}}{2\pi v^2} \left[ \mathbf{n} \times \mathbf{E} + \frac{1}{3m_\psi} (\mathbf{n} \cdot \mathbf{e}_z) \partial_t \mathbf{E} \right], \quad (9)$$

where the stiffness matrix elements are given by  $\rho_s^{ij} = (Z/v^2)[\kappa(\delta_{ix}\delta_{jx} + \delta_{iy}\delta_{jy}) + (\kappa + \sigma_{xy}/(2\pi m_\psi))\delta_{iz}\delta_{jz}]$ , with  $Z = [1 - m_\psi \sigma_{xy}/(2\pi v^2 J)]^{-1}$ . Eq. (9) is one of the main results of this paper. It leads to a fluctuation-induced magnetoelectric effect. One important consequence of Eq. (9) is that due to the fluctuation-induced electric field, the magnitude of the magnetization is not constant, as it would be in the case of absence of Coulomb interaction or for a constant electric field. In particular, if the electric field is only due to external effects, this result implies that we can use a time-dependent electric field to control the magnitude of the magnetization.

Part of the coupling to the electric field, discussed previously by Garate and Franz [9], is reproduced here as a fluctuation effect due to the Coulomb interaction between the spin-orbit coupled electrons lying on the surface of a TI. We have obtained in addition a contribution involving  $\partial_t \mathbf{E}$  that accounts for the time-dependence of the electric field. Note that the term involving  $(\partial_t^2 \mathbf{n})_z$  is typically small at low energy and can be safely neglected in most calculations.

Next we discuss the case of an AF layer on a TI substrate at zero temperature, which, as we will see, differs fundamentally from the case of a FM layer. In the AF case a quantum phase transition occurs at the interface. In order to study the

phase structure of the theory in this case, we will work only in imaginary time from now on. Specifically, we consider an Euclidean effective field theory whose Lagrangian has the form,

$$\mathcal{L} = \bar{\psi}(\not{\partial} - ig_1\not{\sigma} + g_2\sigma)\psi + \frac{\zeta}{2}a_0|\nabla|a_0 + \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\mathbf{a})^2] + \frac{M^2}{2}(\sigma^2 + \mathbf{a}^2) + \frac{\lambda}{4!}(\sigma^2 + \mathbf{a}^2)^2, \quad (10)$$

where we are not making any longer distinction between upper and lower covariant indices, since the metric of the theory has now Euclidean signature. Note that we are also assuming that  $g_1 \neq g_2$ , as quantum fluctuations induces an anisotropy. In the spirit of effective field theories, the coupling constants are understood as effective parameters to be determined by a renormalization group (RG) flow. Thus, the phase structure of the theory is completely determined by the RG equations for the coupling constants.

At low energies and one-loop order (see below) the fixed point structure will be governed by the dimensionless couplings  $\hat{g}_i^2 = g_{i,r}^2/M_r^\epsilon$  ( $i = 1, 2$ ) and  $\hat{\lambda} = \lambda_r/M_r^\epsilon$ , where  $\lambda_r$  and  $g_{i,r}$  are corresponding renormalized couplings and we are using the renormalized mass  $M_r$  as renormalization scale [22]. Here  $\epsilon = 4 - d$ , where  $d = D + 1$ , with  $D$  being the spatial dimension. Our analysis is done in the framework of the  $\epsilon$ -expansion, which is carried out up to one-loop order. As usual, in such a renormalization scheme the renormalized mass gives the inverse of the correlation length, i.e.,  $M_r = \xi^{-1}$ . Due to the coupling between  $\sigma$  and the fermions, a mass anisotropy will be generated, defining in this way two correlation lengths, related to longitudinal and transversal fluctuations. We will assume that  $\xi$  refers to the longitudinal correlation length, giving the fluctuations of the  $\sigma$  field. The correlation length due to transversal fluctuations will be denoted by  $\xi_\perp$ . If  $\nu$  and  $\nu_\perp$  are respectively the critical exponents of the longitudinal and transversal correlation lengths, we easily obtain that  $\xi \sim \xi_\perp^{\nu_\perp/\nu}$ , which determines the crossover exponent  $\phi = \nu/\nu_\perp$ . The quantum critical behavior can be derived from a generalization of the extended Gross-Neveu model [23] discussed in Ref. [24]. We obtain in this way the one-loop  $\beta$  functions  $\beta_{\hat{g}_1^2} \equiv M_r \partial \hat{g}_1^2 / \partial M_r$ ,  $\beta_{\hat{g}_2^2} \equiv M_r \partial \hat{g}_2^2 / \partial M_r$  and  $\beta_{\hat{\lambda}} \equiv M_r \partial \hat{\lambda} / \partial M_r$  in the form,

$$\beta_{\hat{g}_1^2} = -\epsilon \hat{g}_1^2 + \frac{N}{12\pi^2} \hat{g}_1^4 \quad (11)$$

$$\beta_{\hat{g}_2^2} = -\epsilon \hat{g}_2^2 + \frac{N+3}{8\pi^2} \hat{g}_2^4, \quad (12)$$

$$\beta_{\hat{\lambda}} = -\epsilon \hat{\lambda} + \frac{1}{8\pi^2} \left( \frac{11}{2} \hat{\lambda}^2 + 2N \hat{\lambda} \hat{g}_2^2 - 12N \hat{g}_2^4 \right). \quad (13)$$

The  $\beta$  function for  $\hat{\zeta} = \zeta_r/M_r$  follows from the non-locality of the quadratic term in  $a_0$ . Since counterterms are local, this term does not renormalize, which implies simply  $\beta_{\hat{\zeta}} = (N \hat{g}_1^2 / 12\pi^2 - \epsilon) \hat{\zeta}$ . Note that at one-loop order the RG flow for  $\hat{g}_1^2$  decouples from the other couplings.

The quantum critical point is determined by demanding that the  $\beta$  functions above vanish, which yields the infrared stable fixed points,  $\hat{g}_{1*}^2 = 12\pi^2\epsilon/N$ ,  $\hat{g}_{2*}^2 = 8\pi^2\epsilon/(N+3)$ , and  $\hat{\lambda}_* = 8\pi^2\epsilon(3 - N + \sqrt{N^2 + 258N + 9})/[11(N+3)]$ . The anomalous dimension  $\eta_N$  of the Néel order parameter at the interface can be defined via the scaling behavior  $\langle\sigma\rangle \sim M_r^{(2-\epsilon+\eta_N)/2}$ , and is given at one-loop by  $\eta_N = N \hat{g}_{2*}^2 / (8\pi^2) = N\epsilon/(N+3)$ . For  $N = 1$  and two spatial dimensions (corresponding to  $\epsilon = 1$ ), we obtain  $\eta_N = 1/4$ . This large value of the anomalous dimension, as compared to the value obtained from the  $O(3)$  universality class, reflects the fact that  $\langle\sigma\rangle$  receives contributions from the composite operator  $\bar{\psi}\psi$ . The scaling behavior of  $\mathbf{n} = (n_x, n_y, \sigma)$  at the interface is anisotropic, and the transversal fluctuations have a different anomalous dimension, which is dominantly determined by the vacuum polarization diagrams,  $\eta_N^\perp = \epsilon$ , yielding  $\eta_N^\perp = 1$  for  $D = 2$ . It is worth to mention that two-loop corrections will be small, but positive (typically  $\sim 0.03$ ). Therefore, we expect that a more accurate value for the anomalous dimension  $\eta_N^\perp$  is slightly above the unity.

The electrons at the interface also have an anomalous scaling at the quantum critical point, where  $\langle\sigma\rangle$  vanishes at zero temperature and the electrons become gapless. This is in contrast with the FM case, where the fermionic spectrum is always gapped at zero temperature. Thus, we obtain the low-energy behavior,  $\langle\bar{\psi}(p)\psi(p)\rangle \sim -i\not{p}/p^{2-\eta_\psi}$ , where  $\eta_\psi = \hat{g}_{2*}^2/(16\pi^2) = \epsilon/[2(N+3)]$ . For  $D = 2$  and  $N = 1$ , we obtain  $\eta_\psi = 1/8$ . Note that  $\eta_\psi$  does not receive any contribution from the fixed point  $\hat{g}_{1*}^2$  at one loop order. This is due to the fact that the vector field propagator takes here the same form as one in QED where the Feynman gauge has been fixed.

It remains to compute the critical exponents of the correlation lengths. The longitudinal correlation length exponent is given by  $\nu = (2 + \eta_M)^{-1}$ , where at one-loop,  $\eta_M = -5\hat{\lambda}_*/(48\pi^2) - \eta_N$ . Thus, by expanding up to first order in  $\epsilon$ , we obtain,

$$\nu \approx \frac{1}{2} + \frac{\epsilon}{4(N+3)} \left[ \frac{5}{66} (3 - N + \sqrt{N^2 + 258N + 9}) + N \right]. \quad (14)$$

Setting once more  $D = 2$  and  $N = 1$ , we obtain  $\nu \approx 0.649$ . The transversal correlation length exponent, on the other hand, is given by  $\nu_\perp = (2 + \eta_M^\perp)^{-1}$ , where  $\eta_M^\perp = \eta_M - \eta_N^\perp + \eta_N$ . Thus, we obtain  $\nu_\perp \approx \nu + 3\epsilon/[4(N+3)]$ , which for  $N = 1$  and  $D = 2$  yields  $\nu_\perp \approx 0.83$ . Note that the values of the correlation length exponents differ appreciably from the one-loop value of the  $O(3)$  universality class,  $\nu_{O(3)}^{\text{one-loop}} \approx 0.614$ . The correlation length exponents calculated at one-loop are considerably less accurate than the anomalous dimensions, since two-loop corrections usually make significant changes in  $\nu$ . However, our calculation clearly shows the trend towards a novel quantum universality class at the interface of a TI with an AF.

In conclusion, we have shown that the interface between a topological insulator and a magnetic layer exhibits interesting fluctuation-induced magnetoelectric effects. Indeed, we have

shown in the FM case that an axion-like term is generated in the form of a CS term, which in turn modifies the magnetization dynamics of the LL equation. Furthermore, we have shown that for a specific temperature window is possible to have gapless fermions at the interface and, at the same time, a ferromagnetically ordered layer. It would be interesting to check this result experimentally by studying the temperature variation of the surface states.

For the case of an AF layer, we have shown that a quantum phase transition occurs at the interface, and that the fermion spectrum becomes gapless at the QCP. Moreover, large values of the anomalous dimensions for the Néel order parameter were obtained.

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